

UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

LIBRARY ROUTINE F 3 - 129

TITLE	Integration of n Simultaneous Second Order Differential Equations with Initial Conditions Specified (SADOI or DOI)	
TYPE	Subroutine	39
	Parameter storage	$\frac{1}{40}$
TEMPORARY STORAGE	0, 4n locations specified by parameters located in 4 and 5.	
PARAMETERS	3 00 F 00 aF	a is the location of the first word of the auxiliary routine.
	4 00 F 00 (A_0)F	Storage positions A_i , $A_i = A_0 + i$ contain dependent variables y_i ($i = 0, 1, \dots, n-1$). Initial values of y_i must be placed here originally.
	5 00 F 00 ($B_0 - A_0$)F	Storage positions B_i , $B_i = A_0 + (B_0 - A_0) + i$, contain hy_i' where h is the increment. Storage positions C_i , $C_i = A_0 + 2(B_0 - A_0) + i$ are used for temporary storage. Originally B_i and C_i must contain hy_i' and $h^2 y_i''$, respectively. Locations D_i , $D_i = A_0 + 2(B_0 - A_0) + i$ contain values of $h^2 y_i''$ found by the auxiliary subroutine.
	6 00 F 00 nF	n is the number of differential equations to be integrated.
	7 00 F 00 eF	e is the address of the tolerance, ϵ . ϵ is a number such that if, for all i, the absolute value of the difference between two successive approximations to $h^2 y_i''$ is less than $2^{-39} \epsilon$, the iteration process is assumed to have converged.

AUXILIARY SUBROUTINE

The purpose of the auxiliary subroutine is to take the quantities y_i and hy_i' ($i = 0, 1, \dots, n-1$), whose locations are determined by the parameters A_1 and B_1 and compute the quantities $h^2 y''$. The quantity h is the increment of the independent variable. The auxiliary subroutine must be located in a sequence beginning with location a where a is specified by the parameter $S3$. In integrating over one step, the integration subroutine will call in the iterations necessary for convergence.

DURATION

The time in milliseconds for one time through the subroutine is

$$0.54 + 2.17n + I(0.25 + 3.8n + D)$$

where I = number of iterations for an interval

D = time of auxiliary routine

n = number of equations.

DESCRIPTION

This routine will integrate n second order differential equations of the type:

$$(1) \quad y_i'' = f_i(y_0, y_1, \dots, y_{n-1}, y_0', y_1', \dots, y_{n-1}')$$

$$i = 0, 1, \dots, n-1$$

When entered with the entry

p	
	50 p
	26 integration routine,

the subroutine will carry out one integration step replacing values of $y_{i,j-1}$ and $hy_{i,j-1}'$ by $y_{i,j}$ and $hy_{i,j}'$.

THE INDEPENDENT VARIABLE If the independent variable x occurs in the functions f_i , or if it is required during an integration as an index, then it may be obtained by integrating the equation $y'' = 0$. The independent variable x is then treated as an additional dependent variable, for which no additional computing is required by the auxiliary subroutine. If the independent variable does not

appear in any of the f_i 's, but is merely wanted for indication purposes, it is quicker to use a simple counter in the main routine.

REMARKS

The numerical procedure is given by equations 2, 3, and 4.

$$(2) y'_{i,j,k} = y'_{i,j-1} + (h/2) f_{i,j-1} + (h/2) f_{i,j,k}$$

$$(3) y_{i,j,k} = y_{i,j-1} + hy'_{i,j-1} + (h^2/3)f_{i,j-1} + (h^2/6)f_{i,j,k}$$

$$(4) \left. \begin{array}{l} y_{i,j} = y_{i,j,k} \\ y'_{i,j} = y'_{i,j,k} \\ f_{i,j} = f_{i,j,k} \end{array} \right\} \text{ if, for all } i, \text{ the relation}$$

$$h^2 |f_{i,j,k} - f_{i,j,k-1}| \leq \epsilon \quad \text{is satisfied.}$$

In the above equations:

i indicates the variable of integration,

j indicates the step of integration,

and k indicates the number of iterations.

Sufficient conditions to guarantee that the procedure will converge and that numbers stay in bounds can be stated, but use of these conditions leads to extremely short intervals. In practice, if units of measure and a value for h are chosen so that numbers are within bounds within the auxiliary routine the value of h so chosen is usually satisfactory for convergence of the numerical procedure.

DATE	March 18, 1954	rt. 10/7/60
CODED BY	John Brooks	
REVISED BY	Gene H. Golub	
APPROVED BY	<i>J.P. Nash</i>	

LOCATION	ORDER	NOTES		PAGE 1
0	60K(F) K5 1F 42 35L		Plant link	
1	L5 L L4 28L	from 14	(+)1	
2	L0 9L 32 14L		Form $i - (n+1)$ Transfer if $i = n+1$	
3	L4 9L 46 28L		Store i	
4	L4 11L 46 12L		Form A_i	
5	10 20F 42 11L			
6	L4 27L 42 10L		Form B_i	
7	42 13L L4 27L		Form C_i	
8	42 9L 42 12L			
9	L5 S6 L5 ()F	by 8	Dummy instruction used as constant	
10	10 1F 50 ()F	by 7		
11	S4 1022S4 L4 ()F	by 5	Address digits used in computation of A_i	
12	40 ()F L5 ()F	by 4 by 8		
13	S4 F 40 ()F	by 7		
14	26 1L 50 14L	from 2, 36	Enter auxiliary routine	
15	26 S3 47 28L		Set $i = 0$	
16	40 37L L5 L	from 34 from 33	Store , a yes or no mark	

LOCATION	ORDER	NOTES		PAGE 2
17	L4 28L L0 9L		(+)1 Form 1 - (n+1)	
18	32 34L L4 9L		Transfer if i equals n+1	
19	46 28L L4 11L		Store i Form A_i	
20	46 32L 10 20F			
21	42 31L L4 27L		Form B_i	
22	42 29L 42 30L			
23	L4 27L 42 26L		Form C_i	
24	00 20F 46 27L			
25	L4 36L 46 26L		Form D_i	
26	L5 ()F 50 ()F	by 25 by 23		
27	40 ()F 80 S5	by 24		
28	50 ()F 40 F		i by 3, 15, 19, 34	
29	10 1F L4 ()F	by 22		
30	50 F 40 ()F	by 22		
31	7J 38L L4 ()F	by 21	Multiply by equal 1/6	
32	40 ()F L7 S7	by 20		
33	L2 F 32 16L		Question convergence	

LOCATION	ORDER	NOTES	PAGE 3
34	26 16L 47 28L	from 18	Set i equal to zero
35	15 37L 32 ()F	by 0	Iteration finished if equal zero
36	50 S5 22 14L		
37	00 F 00 F		
38	00 F 00 1666 6666 6667 J		